

Lightweight Probabilistic Deep Networks for Cell Segmentation



TECHNISCHE
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Probabilistic Graphical Models: Bonus Project



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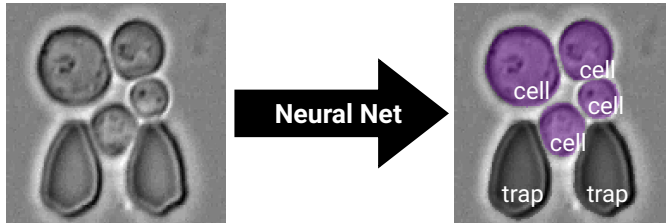


Figure: Trapped yeast cell segmentation example, adapted from [Prangemeier et al., 2020a].

- Goal: Segment cells and traps with uncertainty

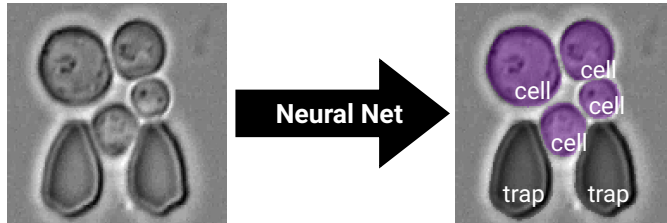


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- Goal: Segment cells and traps with uncertainty
- For the sake of simplicity: semantic segmentation & aleatoric uncertainty



- Bayes by Backprop [[Blundell et al., 2015](#)]
- Probabilistic Backpropagation [[Hernández-Lobato and Adams, 2015](#)]
- Monte Carlo Dropout [[Gal and Ghahramani, 2016](#)]
- Deep Ensembles [[Fort et al., 2019](#)]
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- **Lightweight Probabilistic Deep Networks** [[Gast and Roth, 2018](#)]

- Deep neural network definition (chain of nonlinear layers)

$$\mathbf{y} = \mathbf{f}(\mathbf{x}; \boldsymbol{\theta}) = \mathbf{f}^{(l)} \left(\mathbf{f}^{(l-1)} \left(\dots \mathbf{f}^{(1)} \left(\mathbf{x}, \boldsymbol{\theta}^{(1)} \right) \right) \right)$$

- Each activation is a deterministic point estimate

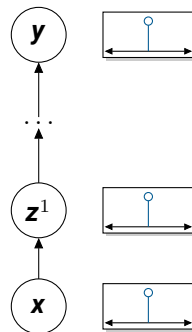


Figure: Deterministic deep neural network.

Lightweight Probabilistic Deep Networks [Gast and Roth, 2018]

Method

- Deep uncertainty propagation using assumed density filtering (ADF)
 - ▣ Assume input to be corrupted by white Gaussian noise

$$p(\mathbf{z}^{(0)}) = \prod_j \mathcal{N}(z_j^{(0)} | x_j, \sigma_n^2)$$

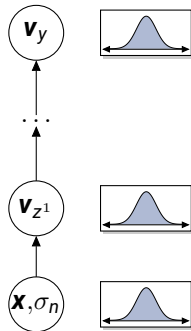


Figure: ADF-based deep neural network.

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- ▣ Use ADF to find a tractable approximation of the network activations

$$p(\mathbf{z}^{(0:l)}) \approx q(\mathbf{z}^{(0:l)}) = q(\mathbf{z}^{(0)}) \prod_{i=1}^l q(\mathbf{z}^{(i)})$$

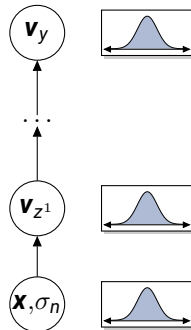


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$$p(\mathbf{z}^{(0:l)}) \approx q(\mathbf{z}^{(0:l)}) = q(\mathbf{z}^{(0)}) \prod_{i=1}^l q(\mathbf{z}^{(i)})$$

- ▣ Approximate subsequent activations by isotropic Gaussian

$$q(\mathbf{z}^{(i)}) = \prod_j \mathcal{N}(z_j^{(i)} | \mu_j^{(i)}, \sigma_j^{(i)}), \mathbf{v}_{\mathbf{z}^{(i)}} = (\boldsymbol{\mu}^{(i)}, \boldsymbol{\sigma}^{(i)})$$

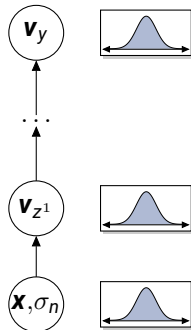


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 - ▣ Transform activation distribution of subsequent layers

$$\tilde{p}(\mathbf{z}^{(0:i)}) = p(\mathbf{z}^{(i)} | \mathbf{z}^{(i-1)}) \prod_{j=0}^{i-1} q(\mathbf{z}^{(j)})$$

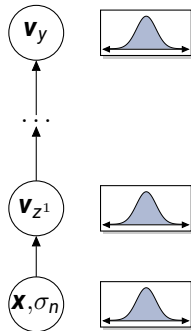


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- ▣ ADF performs incremental updates of variational approximation by solving

$$\underset{\tilde{q}(\mathbf{z}^{(0:i)})}{\operatorname{argmin}} \operatorname{KL} \left(\tilde{p}(\mathbf{z}^{(0:i)}) \parallel \tilde{q}(\mathbf{z}^{(0:i)}) \right)$$

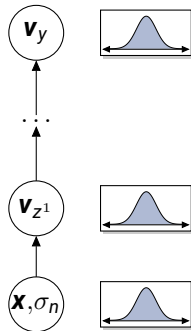


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- ▣ Solve var. approx. by moment matching between $\tilde{p}(\mathbf{z}^{(0:i)})$ and $\tilde{q}(\mathbf{z}^{(0:i)})$.

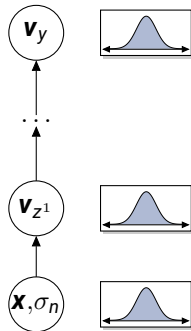


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Method

- Deep uncertainty propagation using assumed density filtering (ADF)
 - ▣ Convert a network layer $\mathbf{z}^{(i)} = \mathbf{f}^{(i)}(\mathbf{z}^{(i-1)}; \theta)$ into an uncertainty propagation layer by simply matching first and second-order central moments

$$\mu_z^{(i)} = \mathbb{E}_{q(\mathbf{z}^{(i-1)})} [\mathbf{f}^{(i)}(\mathbf{z}^{(i-1)}; \theta)]$$

$$\sigma_z^{(i)} = \mathbb{V}_{q(\mathbf{z}^{(i-1)})} [\mathbf{f}^{(i)}(\mathbf{z}^{(i-1)}; \theta)]$$

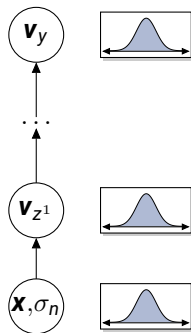


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- ▣ Closed form solution available for the most common layers: Linear, Convolution, Pooling, Upsampling (Trans. Conv.), Leaky ReLU [Gast and Roth, 2018].

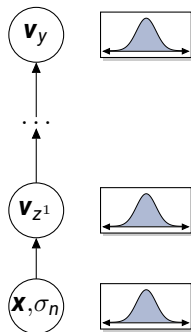


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- Further reading: original paper [Gast and Roth, 2018], [Murphy, 2012] & [Murphy, 2022]

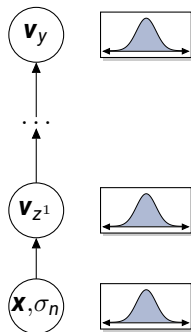


Figure: ADF-based deep neural network.

Lightweight Probabilistic Deep Networks [Gast and Roth, 2018]

Method

- The output of the ADF-based Lightweight Probabilistic Deep Neural Network in the semantic segmentation case is a parameterized Dirichlet distribution for each pixel

$$p(\cdot | \mathbf{z}) = \text{Dir}(\cdot | \alpha(\boldsymbol{\mu}_z, \mathbf{v}_z)), \quad \alpha(\boldsymbol{\mu}_z, \mathbf{v}_z) = \frac{\mathbf{m}}{s}$$

$$\mathbf{m} = \text{softmax}(\boldsymbol{\mu}_z), \quad s = c_1 + c_2 \sqrt{\sum_j m_j v_j}$$

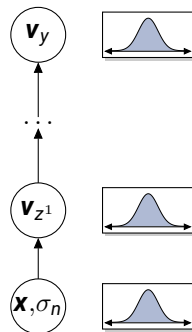


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- Learning is performed by minimizing the conditional negative log-likelihood

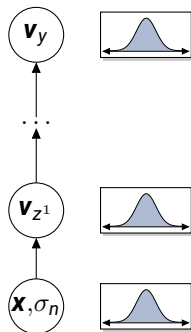


Figure: ADF-based deep neural network.

Table: Numerical results on the trapped yeast cell dataset [Prangemeier et al., 2020b].

Model	Approach	Dice \uparrow	IoU \uparrow
U-Net w/ BN [Ronneberger et al., 2015, Prangemeier et al., 2020b]	Deterministic	0.9626	0.8839
U-Net [Ronneberger et al., 2015]	ADF-based	0.9544	0.9033
DeepLabV3+ [Chen et al., 2018]	ADF-based	0.9492	0.8941

- Surprisingly, U-Net (ADF-based) slightly outperforms DeepLabV3+ (ADF-based).

Experiments

Qualitative Results ADF-Based U-Net

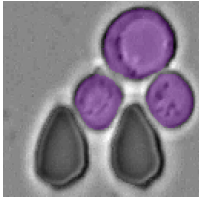


Figure: Segmentation prediction overlay.

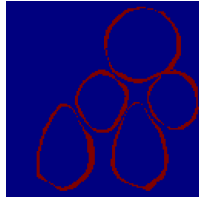


Figure: Misclassified pixels.

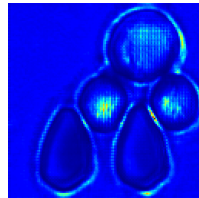


Figure: Mean uncertainty over all classes.

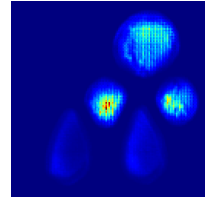


Figure: Background class uncertainty.

- Applied Lightweight Probabilistic Deep Neural Networks to the task of cell semantic segmentation
- Lightweight Probabilistic Deep Neural Networks offer on par segmentation accuracy to the deterministic counterpart while offering uncertainties

Code will be available here:



[https://github.com/
ChristophReich1996](https://github.com/ChristophReich1996)



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